Quiz 01

- Write down your ID number here: _____
- Each item is marked as follows: 0 (not good), 0.5 (so-so), and 1 (good). The maximum total mark is 5. The grade for this quiz is the total mark multiplied by 20.

Suppose $X_1, \ldots, X_n$ are IID normal random variables with mean 0, variance $\theta > 0$, and $E\left(X_1^4\right) = 3\theta^2$.

1. Set up a likelihood function for θ and derive the MLE for θ. Determine whether or not the MLE is also a method of moments estimator of θ.
2. Is the MLE a consistent estimator of θ? Show why or why not.
3. Is the MLE an efficient estimator of θ? Show why or why not.
4. Is $\widetilde{\theta} = X_1^2$ sufficient for θ? Show why or why not.
5. Find the form of the generalized likelihood ratio test for the null that $\theta = 1$ against the alternative that $\theta \neq 1$.