Hints and partial solutions (along with suggested modifications to exercises for more practice)

Exercise 02

Change to $\mathbb{P}(796.2 < \sum_{i=1}^{16} (X_i - \bar{X})^2 < 2630)$.

Exercise 04

Make sure to describe the parameters and what they mean.

If you use an approach looking at means:

$H_0: \mu = 2.5$ (make sure to describe what μ here represents)

$H_1: \mu \neq 2.5$

$\frac{2.13 - 2.5}{\frac{1.14}{\sqrt{30}}} = -1.78$ is the test statistic (observed).

p-value $= 2 * \mathbb{P}(Z < -1.78) \approx 0.075$ where $Z \sim N(0,1)$

Thus, we fail to reject H_0 at 5% level.

This approach is justified only when 30 is large enough, (so that CLT applies) the observed outcomes form an IID sequence.

If you use a goodness-of-fit testing approach

$H_0: p_{10} = \frac{1}{4}, p_{20} = \frac{1}{4}, p_{30} = \frac{1}{4}, p_{40} = \frac{1}{4}$ (make sure to describe what p_{i0} means)

H_1: at least one of the equalities is not true. ($p_i \neq p_{i0}$ for at least one i)

You can use Ex 10.3.1 as a shortcut (and the proportions are all equal, so things become easier)

Test statistic $= 5.467$

Critical χ^2_3 value $= 11.07$

$\Rightarrow$ fail to reject H_0.

You can try finding the p-value (or bounds?)

p-value should be larger than 0.1

Since $\mathbb{P}(\chi^2_3 > 6.251) \approx 0.1$.

Exercise 05

Pay attention to shortcuts here as well!

$$D = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(X_{ij} - n\hat{p}_i\hat{q}_j)^2}{n\hat{p}_i\hat{q}_j}$$

→ also try ~~something~~ to obtain something similar to Ex 10.3.1.

$$= \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(n\hat{p}_{ij} - n\hat{p}_i\hat{q}_j)^2}{n\hat{p}_i\hat{q}_j}$$

$$= n\sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(\hat{p}_{ij} - \hat{p}_i\hat{q}_j)^2}{\hat{p}_i\hat{q}_j}$$

For the exercise, $\sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(\hat{p}_{ij} - \hat{p}_i\hat{q}_j)^2}{\hat{p}_i\hat{q}_j} \approx 0.0068$. (observed D)

Observe that $\mathbb{P}(\chi^2_2 \geq 4.605 \mid H_0 \text{ is true}) = 0.1$.

find out how! (the 2)

So if $0.0068n \geq 4.605$, we should be able to reject null at the 10% level.

Therefore, $n \geq 677$. So 677 is the smallest sample size for which we will be able to reject H_0.

Observe that a large enough sample size should make any discrepancy between what we observe and what we expect to see statistically significant.

Try thinking about what happens if you did not have to estimate p_i and q_j. What happens to the smallest sample size? What does this imply about the ability of goodness-of-fit tests to ~~det~~ detect misspecification even if there is no misspecification?

Exercise 06 (You are not asked to fill up the table!)

1. There are four brands of spark plugs . . .
2. Use the fact that $MSE = \dfrac{SSE}{\text{number of obs} - \text{number of groups}}$

Calculate the F-stat = 1.7

To approximate p-value: ~~Our~~ The tables in the textbook ~~is in~~ are incomplete. You need to look at the patterns. What happens to the probabilities if you change the denominator df? You may need to provide a range for the p-value.

3. You will not reject the null in (2). It is not possible to fulfill the client's request because you ~~cannot~~ do ~~are~~ not have the sample means and sample standard deviations for each brand. ~~You also~~

Exercise 07

This exercise is slightly tricky. But the distribution of ~~is the same~~

$$\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(X_i-\bar{X})^2 + \frac{1}{\sigma_2^2}\sum_{i=1}^{m}(Y_i-\bar{Y})^2$$

will be the same across the different settings.
Try to find out why and to find the ~~dis~~ common distribution!

Exercise 07 is a bit different from the discussions in ANOVA and ~~the~~ Section 9.2 because we are not looking at the distribution of $\dfrac{\bar{X}-\bar{Y}-(\mu_X-\mu_Y)}{\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}}}$

or the ~~F-statistic~~ statistic $\dfrac{SSTR/(k-1)}{SSE/(n-k)}$