In-class exercise, 2023.05.09

- Write down your ID number here: _____
- Write down your partner's ID number here: _____
- The grading is as follows: If you did not even try or are doing something else instead of this exercise, then you get 0 automatically and will be marked absent.

Let Y_1, Y_2 be IID $N\left(\mu, \sigma^2\right)$.

1. Write down the joint density of Y_1, Y_2. After finding the joint density, verify that the resulting density is a specific case of LM Definition 11.5.1.
2. We will change variables from Y_1, Y_2 to V_1, V_2 where $V_1 = \frac{1}{\sqrt{2}} Y_1 + \frac{1}{\sqrt{2}} Y_2$ and $V_2 = \frac{1}{\sqrt{2}} Y_1 - \frac{1}{\sqrt{2}} Y_2$.
   (a) Solve for Y_1, Y_2 in terms of V_1, V_2.
   (b) Substitute the result in (a) into your joint density in Item 1. Do some algebra and try to obtain a simplified form similar to LM Definition 11.5.1.
   (c) Is your simplified form in (b) a joint density? If it is, what is the distribution of V_1, V_2? If it is not, what should you do so that it would become a joint density?